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ILLEGITIMATE TENDENCIES IN THE USE OF THE CONCEPT OF SELF-SIMILAR PHENOMENA *

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In 1978, Gidrometeoizdat published a book "Similarity, Self-similarity, Intermediate Asymptotics," by G. I. Barenblatt of 206 pages with a foreword by Academician Ia. B.Zel'dovich.

The author's purpose is not to give any thorough exposition of the state of the art of questions from mechanics, physics or engineering. The purpose of the book is to present mathematical illustrations for the foundations of some, as will be shown below, inconsistent assertions, to study the methodological nature and realizations of the tendency to introduce invented myths about the achievements of some authors with the disparagement of the value and meaning of the results of other authors. All this is performed in a background of numerous errors in fact and inadequate discussions and assertions on the crux of the matter.

The main basic idea of the book is to attempt to separate self-similar solutions of partial differential equations into solutions obtained from "naive considerations of dimensional analysis" (p.145), which is the solution of the first kind, the "visible part of the iceberg" (pp.8, 51), and solutions of the second kind, which are actually important and nontrivial to the author, and which "were explicitly isolated into a special class" by Akad. Zel'dovich, who expressed enthusiasm about the book in the foreward by calling it a source of inspiration.

By using dimensional analysis, L. I. Sedov indicated methods in the theory of self-similar processes that permit giving answers to the following questions:

1°. A formulation is given for a problem (i.e., all the equations are given and all the supplementary conditions governing the solution in a theory or all the essential and admissible conditions in an experiment). Will the phenomenon under consideration be self-similar?

2°. What properties should the formulation of physical problems possess so that their solution would be self-similar?

It is also useful to emphasize that these results are given a foundation in the theory by not only the study of the properties of the appropriate equations but also by an analysis of the explicitly formulated supplementary conditions: initial and boundary conditions, relationships on strong and weak discontinuities, and conditions of other kinds.

Also given are derivations of expedient methods to process experiments that permit obtaining universal curves, for instance in the nonself-similar problem of a point explosion with counter-pressure taken into account, etc.

On the basis of this theory, Sedov gave a formulation and the solution of a large number of new problems that have self-similar solutions. He afterwards (in 1945-1946) derived all the self-similar solutions for the one-dimensional unsteady motions of a perfect gas with an appropriate system of strong discontinuities present in the flow,which generally move at variable velocities.

Many self-similar solutions on different special problems were already known prior to the work of Sedov. Many such examples are described in his book "Similarity and Dimensional Analysis in Mechanics" (first edition in 1944). Self-similar solutions were also known for one-dimensional unsteady gas motions. For instance, for detonation problems in particular, the self-similarity of the spherical detonation problem was revealed by O. E. Vlasov in 1937. Centered Riemann and Prandtl—Meyer solutions, the solution of the problem of diffusion of a rectilinear vortex in a viscous fluid, the solution of the Boussinesq problem in elasticity theory, self-similar solutions in the theory of isotropic turbulence, and many other self-similar regularities have long been known.

K. Bechert also studied particular forms of self-similar solutions for gas motions, which did not, however, permit setting up the problems and finding their solutions in the presence of strong discontinuities in the flow. For motions with spherical and cylindrical symmetry, Bechert considered only continuous motions in the presence of barotropic polytropic processes.For the equations of ideal gas motion with plane waves he mentioned particular examples of a solution with a given entropy distribution in particles, where the gas particle velocity in these solutions depends linearly on Cartesian coordintes. He did not apply his formula to the solution of any problems.

For the full-fledged application of group-theory methods to seek solutions of certain problems, it is necessary to verify not only the invariance of the equations relative to

*Prikl.Matem.Mekhan.,44,No.2,369-377,1980

appropriate transformation groups, as Bechert did, but also the invariance of the additional conditions which extract the desired solutions that are of self-similar form. There is no such operation in the Bechert researches.

In this connection, we cannot speak about the identity of the transformation groups cited only for the equations and the appropriate analysis of the formulation of problems by using dimensional analysis.

A paper of Guderley (*) was published in Germany in 1942 in which he examined the special question of the approximate asymptotic law to solve the problem that he formulated of adiabatic gas motion with variable entropy in particles near the center of symmetry in the spherical and cylindrical cases, in the approximation that the corresponding asymptotic laws are represented by series whose first terms are self-similar solutions according to modern terminology, and he moreover obtains only one particular solution in each kind of symmetry. The general property of physical problems whose solution is self-similar is missing from this Guderley paper.

It should be added that the problem Guderley studied was not examined in the L. I. Sedov researches, and all the problems posed, studied, and solved efficiently by Sedov were not examined by Guderley. As regards the derivation of the appropriate ordinary differential equations and the conditions on the shocks, such derivations are typical, elementary, and of no value in principle. L. I. Sedov gave the appropriate notion, equations, and qualitative schemes of investigation in the general case (and not in the particular case as Guderley did) in especially simple form; they are convenient and indeed they are standard in the world literature and, particularly everywhere in the book under discussion and in the other publications of Barenblatt.

It is important to characterize more definitely the analogous equations and the method of obtaining them in the paper of Guderley. Firstly, he examines only the particular self-similar solution that corresponds to $k = -3, s = 0$ and $n = \delta$ in the notation of Sedov; secondly, Guderley converts some dimensionless ordinary differential equations (7a), (7b) and (7c) by complex means to other also dimensionless differential equations (13a) and (14). Such a conversion is evidently not related substantially to dimensional analysis although Guderley did indeed try to give this transformation a foundation by relying on dimensional analysis, and only thereby does he apply dimensional analysis! Consequently, he finds the self-similarity exponent n approximately for the unique solution he obtained (**) by a mathematical analysis of the field of integral curves for equation (13a) (for the spherical case $n = \delta = 0.717$).

It must still be emphasized that the self-similar form of the solution was assumed by Guderley in his problem, but was not proved, and he nowhere substantially uses the methods of dimensional analysis, and his remark about the self-similarity of the solution is just a statement of the consequence of his assumption (not the proof) of the mathematical nature of the desired principal term of the approximate asymptotic in the problem of describing the reflection of a spherical shock from a center.

It is impossible to ascribe credit to the authors of the important and interesting papers of Bechert and Guderley for the creation of a general theory of self-similar phenomena in gas dynamics nor in different applications of mathematics and physics. Many other authors who solved particular self-similar problems predetermined the general theory of self-similar phenomena by their contributions to no less a degree than did Bechert and Guderley. It is sufficient to recall the researches of Riemann, Prandtl, and Meyer, Rayleigh, Busemann, Kármán, Boussinesq, and many other authors.

It follows from the above that the attempts of Barenblatt to resolve priority questions as he desires (pp. 70, 82) despite the facts are without any foundation, and one of the fundamental theses of the proposed criticism is thereby illustrated.

In 1945, K. P. Staniukovich formally considered the self-similar equations of one-dimensional unsteady gas motion when the entropy can have different values on different particles by using the appropriate substitutions of formulas for the kind of desired functions in the equations without using the methods of dimensional analysis (exactly as Guderley did).

Thus, in gas dynamics Barenblatt uses and discusses just those systems of ordinary equations and just those of their solutions that had already been given and studied by Sedov; hence all the solutions corresponding to the underwater part of the "iceberg" are already contained in this family.

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*) Guderley, G., Starke kugelige und zylindrische Verdichtungsstösse in der Nähe des Kugelmittelpunktes bzw. der Zylinderachse. Luftfahrtforschung, Bd. 19, H. 9, S. 302-313, 1942.

**) Guderley obtains a unique solution for the $\varkappa$ he selected in the formulation of his particular problem $(k = -3, s = 0)$ but papers published later are well known, wherein numerous asymptotic solutions of gas motions of convergent and divergent types are studied near the center of symmetry, for which the Guderley assumptions are not satisfied.

As is known, it is sufficient to give the dimensions of the determining parameters in order to extract a certain specific solution or whole family of self-similar solutions with a given self-similarity exponent when self-similar solutions exist. In particular, transcendental numbers are included in the dimensions of the essential parameters, whose exponents can be arbitrary.

The assertion contained also in the foreword of Zel'dovich that the type of self-similarity in problems of the "first kind" is characterized by given dimensional exponents that are represented by "simple fractions" is quite curious. To refute this assertion it is sufficient to refer Barenblatt and Zel'dovich to the problem of a piston (*) with the law of motion $x = kt^{\pi/4}$, $[k] = LT^{-\pi/4}$($\pi = 3.14\ldots$ being a known transcendental number). Self-similarity is assured by the initial condition that the pressure is zero in a homogeneous, ideal, perfect gas at rest.

Barenblatt tries to qualify self-similar solutions of those problems in which the dimensions of the determining parameters are given beforehand as solutions of the first kind — these are the "naive" solutions — and those solutions in which the dimensions of the determining parameters are not given explicitly but can substantially be determined from the mathematical conditions of the problem, as solutions of the "second kind", for example, the solutions and appropriate dimensional parameters corresponding to the "model" problem of taking account of radiation in a strong explosion.

However, Barenblatt does not remark, or avoids discussing the fact that in each of the problems of the "second kind" that he considered it is also possible, following typical examples of formulations of problems with solutions of the first kind, to give the dimensions of the determining parameters first in the most "naive" manner and to give thereby the value of a certain exponent α and then to determine the dimensionless parameters in the boundary conditions, for instance γ_1 in the problem of an explosion with energy loss because of radiation at the shock wave (**) or in the extraction of energy at a wave front of detonation type (the Oppenheim solution of this same problem for $\gamma_1 > \gamma$, which Barenblatt cites, is from 1970), or when the ratio $\varkappa_1/\varkappa \neq 1$ in the problem of fluid filtration in underground strata, etc.

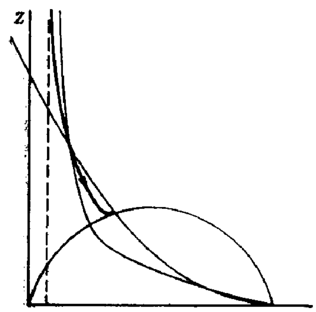

Fig.1

A function $\gamma_1 = f(\alpha, \gamma)$ is easily constructed by such means in the problem of taking account of radiation in explosions or $\varkappa_1/\varkappa = F(\alpha)$ in the problem of filtration and in the problem of heat conduction with discontinuous specific heat. The possibility is afterwards manifest of answering all questions related to the problems under consideration. It is remarkable that he is indeed obliged to proceed in an analogous manner to a factual description of the construction of the solutions of problems listed above when the construction of the self-similar solution is performed by selecting α after which the desired reformulation of their statement is given.

In this connection, it is still useful to add that Barenblatt does not cite the solution of the problem of a detonation with energy liberation proportional to the coordinate of the front to some power $(Q = Q_0 r^{2m})$, which is completely equivalent to energy liberation in proportion to the temperature behind the discontinuity (which is equivalent to $\gamma_1 > \gamma$), solved by Ia. G. Sapunkov as a problem of the "first kind" in 1967, who found such solutions because of his "naive approach", the major part of which were not noted by Barenblatt. On p. 74 of his book, the "portrait" of the integral curve field is incomplete, and he has omitted an entire continuum of values of α: $(3\gamma + 3)/(5\gamma + 3) < \alpha < 1$, $\gamma_1 = 2\gamma + 1$, corresponding to solutions about divergent detonation waves of Chapman—Jouguet type, which were described by Ia. G. Sapunkov (***). The appropriate integral curve, missing from the analogous Fig.4.3 in the book, is shown by the heavy line in the Fig.1.

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*) See Krasheninnikova, N. L., "On unsteady motions of a gas displaced by a piston," Izv. Akad. Nauk SSSR, Otdel. Tekhn. Nauk, No.8, 1955, and Grigorian, "Cauchy problem and problem about a piston for one-dimensional unsteady gas motions (self-similar motions)," PMM Vol.22, No.2, 1958.

**) The proposed treatment of the model problem of an explosion with energy loss on an explosive wave front cannot pretend to be a practically satisfactory description of the actual phenomena. Methods of computing explosions with radiation taken into account are well developed in application to actual phenomena, and are published; they have nothing in common with the problem considered by Barenblatt and Zel'dovich, which is discussed here and later.

***) See Sapunkov, Ia. G., "Converging detonation waves in the Chapman—Jouguet regime in a medium with variable and constant initial densities," PMM Vol.31, No.5, 1967.

The main idea of this remark is not related so much to the absence of a reference to Sapunkov as to the feature of the method of analysis taken, which can result in the loss of entire ranges of solutions.

When applying the mentioned algorithm with preassigned indices α, there is obtained by definition that the solution found is a solution of the "first kind" with suitable functions $\gamma_1 = f(\alpha, \gamma)$ and $\varkappa_1/\varkappa = F(\alpha)$ found, but this same solution should be considered, the author proposes, as a solution of the "second kind" for given γ_1 or $\varkappa_1/\varkappa$ and the desired α to be determined. So what is the sense of an imposed classification in which substantially the identical solution is both a solution of the "first" and the "second kinds"?!

Then perhaps a solution of the "third kind" could still be introduced by the same means if strongly desired; this would be those solutions in which the dimensions of the determining parameters for the asymptotic law are found from experiment; for example, such is the situation in investigations of asymptotic laws for the decay of turbulent fluid motions, studied in 1944.

It is evident that in the general case, in the construction of models of real phenomena occurring in nature or in technology, any assumptions that are either "naive" or conversely "pseudo-scientific" should be confirmed by experiments; and in this sense any formulations can be satisfactory only as approximations with known assumptions, and for a limited range of the determining variables that is confirmed by experiments.

From this viewpoint, all self-similar motions relating to reality should be considered as solutions of the "third kind"!

Besides the above, let us note explicitly and emphasize that seeking the "eigenvalue" of the exponent α requires reliance on an additional condition that should be formulated separately in setting up the problem, and which can vary. For instance, an additional condition of continuity of the desired function and its first derivative at the point of discontinuity of the coefficient $\varkappa$ is advanced in the filtration or heat-conduction problem. This condition is physical in nature. It can be taken formally or replaced by another condition on the discontinuity with a physical foundation.

The author refers to S. L. Kamenomostskii, who proved that the equation under consideration has a unique solution of the Cauchy problem under the condition of continuity of the solution at the point of discontinuity of the coefficient $\varkappa$, but this does not eliminate the question about obtaining physically correct solutions that are discontinuous at this point.

The condition on the discontinuity can be a source for determining α, which can (and should) be obtained differently than in the case of the continuous solution. Thus solutions with different given α can be considered in the presence of discontinuities whose nature is determined by the selected values of α.

The story of the solution of the problem of a "short shock", which has long been persistently ascribed to Zel'dovich is not without interest. The first publication containing the solution of the problem of a "short shock" apparently belongs to Weizsäcker and dates from 1954. Then other German papers dealing with this same self-similar problem was published in 1954 and 1955.

V. B. Adamskii and Zel'dovich published the solution of the "short shock" in the same number of the Acoustics Journal in 1956 with the same access date. Such a coincidence in the results obtained by several authors and even, in different countries often occurs in scientific life, but the paradoxical fact here is that this solution is exotic in nature; the corresponding one-dimensional gas motion with plane waves has infinite energy and zero momentum. The last two properties follow trivially from dimensional analysis if attention is turned to the dimensions obtained for the defining constant. It follows from the numerical value of the exponent in the dimensions of this constant that the total energy and the total momentum can be either zero or infinity. Since the energy differs from zero, it is then infinite, and since the appropriate integral converges, the momentum cannot be infinite, hence it is zero. This then is the whole proof showing the wastefulness of the long discussion about this in the paper of Zel'dovich and in the book of Barenblatt.

On p. 46, Barenblatt writes, perhaps naively, in a discussion of the method of solving the problem of a strong explosion that the determination of the principal constant "is in principle completely analogous to what was done in the preceding problem." But the crux of the matter is that the preceding problem of a strong thermal wave was solved somewhat later by Barenblatt and Zel'dovich, where the problem of a thermal wave is much simpler and more elementary.

We make the following important remark relative to the problem of an explosion with radiation.

The author proves that by introducing an energy efflux for a change in the boundary condition on the shock, the limiting self-similar problem with the determining parameters ρ_0, E, γ and γ_1 has no solution because of "radiation" when the domain in which the energy E is liberated shrinks to a point (*).

In fact, it follows from the formulation of the limiting problem that the energy "being emitted" at the front in time dt should be represented by the formula (4.27) (see p. 75 or p. 69)

$$d\varepsilon = -4\pi r_f^2 D \frac{(\gamma-\gamma_1)}{(\gamma-1)(\gamma_1-1)} \rho_0 \frac{p_f}{\rho_f} dt \qquad (1)$$

If the determining parameters r, t, E, ρ_0 and $\xi_0 \neq 0$ (see (4.6), p. 67), then we have

$$r_f = \xi_0 \left(\frac{Et^2}{\rho_0}\right)^{1/5}, \quad D = \frac{dr_f}{dt} = \frac{2}{5} \xi_0 \left(\frac{E}{\rho_0}\right)^{1/5} t^{-3/5}, \qquad p_f = \frac{2}{\gamma_1+1}\rho_0 D^2, \quad \rho_f = \frac{\gamma_1+1}{\gamma_1-1}\rho_0$$

We obtain after substituting these formulas into (1)

$$\frac{d\varepsilon}{dt} = \frac{64\pi}{125} \frac{(\gamma-\gamma_1)}{(\gamma-1)(\gamma_1+1)^2} \xi_0^5 E \frac{1}{t}$$

Hence it follows that as $t \to 0$ for $\xi_0 \neq 0$ the total energy emitted is infinite. However, if the liberated energy is finite by the assumption of the problem, then this means that $\xi_0 = 0$, and hence it follows from the formulation of the problem that in the limit $r_f = 0$ and the whole finite energy is emitted instantaneously and the gas remains unperturbed.

Actually, upon fixing the magnitude of the total energy being liberated during an explosion and diminishing the size of the domain where the energy is liberated, the fraction of the energy going out into a different kind of radiation is raised, and the perturbed gas motion is weakened. This effect is aggravated if the assumption is made (made by Barenblatt and Zel'dovich and actually unacceptable) that the emitted energy is not absorbed ahead of the shock front.

Thus, the limiting self-similar motion reduces to rest, and all the energy is instantaneously emitted. This "motion" is a solution satisfying all the conditions of the formulated problem and it is evident that this solution will correspond qualitatively to reality in the limit.

The proof of Barenblatt (on pp. 68, 69 in the book and in previous papers) of the nonexistence of a solution of the self-similar problem is wrong since it cancels the factor on the right and left which vanishes under the real condition on a shock.

It is curious to note that this error had already been indicated to Barenblatt and Zel'dovich in the press in 1970 (see Ref. Zh. Mekhanika, 10B, 157, 1971); this error was contained in papers published prior to the review in the review journal (**) and after this review(***).

Despite this, and the known effects published in the literature on elevated radiation used with the reduction in mechanical perturbations in atomic explosions, the two authors mentioned did not look into the crux of the matter, and continue to persist in their erroneous derivations in the literature. The asymptotics of solutions of non-self-similar problems examined in the book which deal with unsteady motion of a gas with radiation at the wave shock for $\gamma_1 \neq \gamma$ and $p_0 = 0$, is meaningful only for $x \to \infty$ and $t \to \infty$. The asymptotic solution of Riemann corresponds to the nonself-similar problem without radiation of a point explosion ($p_0 \neq 0$) as $t \to \infty$ and $x \to \infty$. Here the condition $p_0 = 0$ ahead of the shock is conserved even in the presence of radiation in the asymptotic solution as $t \to \infty$.

Thermodynamically correct formulations of physical problems are not given in the book: that is the situation for the problem considered above of an explosion taking radiation into account and for the problem of heat conduction with a discontinuous coefficient of specific heat; in this latter case the internal energy of the medium turns out not to be a unique function of the arguments: the temperature T and the sign of $\partial T/\partial t$ introduced in the formulation of the problem.

Although the crumpling of the porous soil structure is taken into account in the problem of underground fluid filtration in a porous medium with plane waves yet no attention is turned

*) It is clearly formulated on p. 69 that "...the contradiction thus obtained proves the nonexistence for $\gamma_1 \neq \gamma$ of a solution of our problem having the form (4.6)," and nothing is said about the trivial solution, which is not only perfectly reasonable but also reflects the crux of the matter.

**) Barenblatt, and Sivashinskii, Self-similar solutions of the second kind in the problem of propagation of intense shock waves PMM, Vol.34, No.4, 1970, and Barenblatt, and Zel'dovich, Intermediate asymptotics in mathematical physics, Vol.26, No.2, 1971. (English translation in Russian Math. Surveys).

***) Barenblatt, and Zel'dovich. Self-similar solutions as intermediate asymptotics. Ann. Rev. Fluid Mech., Vol.4, 285—312, 1972.

to the dependence of the filtration factors on the porosity and on the possibility of total crumpling for which the porosity vanishes (or can even become negative in the solutions being constructed (!), which strongly influences the motion of the fluid being filtered that cannot at all be unlike the motion which the author obtained in the specific problem considered after the initial rapid sampling of the fluid. Quite important effects due to settling of the soil and plugging up of the porous channels caused by evacuation of the fluid are known from practice.

Nevertheless by rejecting doubts about the physical legitimacy of the considered pretensions from the viewpoint of practical applications of problems of heat conduction and of filtration of fluid, as well as by taking account of radiation during strong explosions, it is possible to be occupied with a discussion of the purely mathematical questions of seeking solutions and their properties for problems formulated with mathematical formality.

From the mathematical viewpoint, in the linear formulation used by the author (linearized equation, see p. 53 in the book), his assertion that the self-similar problem of the removal of a finite mass of fluid at a point in filtration has no solution, is false. A trivial solution with the absence of perturbations is obtained in the formulation. The situation here is the same as in the problem considered above of taking account of radiation, where a solution also exists, but is trivial. Trivial solutions are real and unique solutions, which are obtained as a result of the problem formulation use, and it is not possible to say that they do not exist. Moreover, it can be said that such trivial solutions correspond to reality if it is considered that the formulation of the problems being proposed is acceptable in some sense.

An entirely different matter is the mathematical question of the asymptotics of nonself-similar solutions and of their approximation for large t by means of self-similar solutions with the very same mathematically formulated boundary conditions on the shock in the problem of an explosion with radiation or at a point of discontinuity of the coefficient in the equation describing filtration of a fluid.

Numerical computations of nonself-similar problems and their comparison with corresponding computations of self-similar solutions are given in these two mathematical examples. The results of these computations exhibit a good approximation of the self-similar solutions by the nonself-similar solutions as time elapses. These results are a redundant demonstration of the concept already known for a long time from the very beginning of the development of the theory of self-similar processes, of the role of self-similar solutions as an approximation for nonself-similar solutions, which has been clearly formulated and used in numerous applications.

It was essential in the specific computed examples in the book that the conditions on the shock or the continuity condition for the derivative at the point of discontinuity of $\varkappa$ (the coefficient in the equation $\partial u/\partial t = \varkappa(\partial^2 u/\partial x^2)$) be governing for the asymptotic behavior of the solution. The conditions at the point of discontinuity of $\varkappa = \lambda/c_v$ evidently depend on whether λ or c_v undergoes a discontinuity, or both.

Therefore, in the light of the remarks made above, the mathematical treatment of the problems considered cannot be an occasion for the foundation of any new viewpoint on the meaning of self-similar solutions.

Let us yet add that the principal question in the problems considered:do the constructed solutions yield some "intermediate asymptotic" for actual phenomena?, has received no foundation. Numerical computationsof fluid filtration in soils and of taking account of radiation in an explosion remain without comparison with experiment or with other already available more detailed and physically well-founded theories.

Let us now turn to some separate remarks.

In the light of existing theories, the derivations of the properties of the L. G. Loitsianskii integral in the theory of isotropic turbulence, neglecting or taking account of third-order moments, which were set up by Sedov in 1944, are quite essential. However, this circumstance is not cited and the impression is created that the results mentioned are those of the author of the book himself.

Also not elucidated and not cited are the quite thorough and successful researches of A. I. Korneev, in which he performed a detailed analysis of the published experimental data and established the essential position about the agreement between existing theory and experiment, which was long subjected to unfounded doubts.

Turning to the remarks made by the author himself about the Stewart test data, and the analysis he knows of the Stewart tests, his deduction about the absence of self-similarity for third order moments is hasty and unfounded.

In elucidating the hypotheses of local or complete isotropy of turbulent motions, it is impossible to bypass existing experimental data that directly contradict such hypotheses.

In particular it is found (*) that there is no complete isotropy behind grids since the mean values of the longitudinal and transverse pulsations are different.

Existing theoretical data about laws for the correlation coefficients are missing in the elucidation of the theory of isotropic turbulence with third-order moments taken into account.

Let us note several more specific characteristic errors to illustrate the style used in the book.

The plane problem of potential flow past an infinite wedge with an apex angle of 2α of an ideal incomprssible fluid occupying the whole space outside the wedge is considered, where the "flow has the velocity U at infinity" directed along the wedge axis. Upon introduction of polar coordinates r, θ with center at the wedge vertex, the system of governing parameters U, r, θ, α is written down.

The author later tries to construct the kinematic solution of the problem formulated in this manner and to resolve the "contradiction" that, in his opinion, occurs; however he does not note nor remark the important circumstance that the corresponding fluid motion generally does not exist for $U \neq 0$ and $U \neq \infty$. Hence, there are no "contradictions".

If it is assumed that $U = 0$, then we find that the whole mass of fluid is at rest, but the asymptotics obtained becomes pointless. If $U = \infty$, then the asymptotic self-similar motion near the apex is not determined uniquely without essential additional hypotheses.

The sentence "the contribution of viscous drag in a rough first approximation can be considered small for a ship of good streamlined shape", is contained on p. 34. This assertion is erroneous since the principal part of the hydrodynamic drag (more than 80%) for ships with good streamlined shape is the viscous friction drag. The situation in this passage is not only the wrongness of such an assertion but also that the motivation of the need to simulate Froude tests of ship models is wrong. As is known, such simulation is not related to the smallness of the viscous friction drag.

Particular results are contained in the discussions on pp. 101 and 102, which do not allow the assertion that rarefaction waves are generally impossible, although the author essentially makes that assertion.

Some portion of the book is a compilation of expository material published in other books or in separate papers which Barenblatt thought interesting to cite.

The above is not, by far, a complete listing of all the errors and inadequacies but does characterize the quality of the book completely. It must also be noted that study of this book by young or scientifically immature persons can instill them with an incorrect comprehension of the problems and substance of the situation associated with the theory of self-similar phenomena, and the sence of asymptotic regularities already obtained by other authors.

The present paper is needed to counter the distortions in the already widely propagated theories which have many important practical applications.

Translated by M.D.F.

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*) See G. Compte-Bellot and S. Corrsin, The use of a contraction to improve the isotropy of grid-generated turbulence, J. Fluid Mech., Vol.25, No.4, 657-682, 1966 and G. K. Batchelor and R. W. Stewart, Anisotropy of the spectrum of turbulence at small wave numbers, Quart. J. Mech. Appl. Math., Vol.3, No.1, 1950.